ECON 440: PUBLIC ECONOMICS
FALL 2011
Assignment #1

Due date: October 13th, before 4:00 pm in the 440 assignment box.

1. Consider an exchange economy with two individual, A and B, and two goods, x and y. The preferences of the individuals can be represented by:

$$u^i(x_i, y_i) = \frac{1}{2}\ln x_i + \frac{1}{2}\ln y_i \quad for\ i = A, B.$$

A is endowed with $\omega_A = (\omega_A^x, \omega_A^y) = (4, 1)$ and B with $\omega_B = (0, 1)$.

   a) Write and solve the individual's problem. Find the demand function for each good for both individuals (i.e $x_i(p_x, p_y), y_i(p_x, p_y)$ for $i = A, B$). Then solve for the competitive equilibrium prices and allocation (I suggest you normalize the price of good 1). Then display the equilibrium in an Edgeworth box.

   b) Is the above competitive allocation efficient? Justify your answer using the planner's problem seen in class.

   c) Suppose a government has the following social preferences:

$$W(u^A, u^B) = \min\{u^A(x_A, y_A), u^B(x_B, y_B)\}.$$

   Find the feasible allocation that maximizes the government's preferences. Find the equilibrium price ratio (or just one normalized price) at that specific allocation. Finally, find the value of lump-sum taxes/transfers T^i that is needed to decentralize this allocation.

2. Consider a society with $N > 1$ identical individuals who have the following preferences:

$$u_i(x_i, H) = x_i + \ln H,$$

where x_i is the amount of an aggregate consumption good and H is the amount of public health supplied for the whole society (i.e. vaccines, sanitation services, pandemic surveillance etc.). Furthermore, all individuals have an income of I.

   a) Suppose that public health is produced by a charitable organization that collects individual contributions h_i and produces $\sum_{i=1}^{N} h_i = H$ amount of public health with these contributions. Set up and solve the individual's problem and find its reaction function $h_i^* = h_i(H_{-i})$, i.e. its contribution as a function of all the other individual's contribution ($H_{-i} = \sum_{j \neq i} h_j$).

b) Find the Nash equilibrium amount of public health H^n supplied for society and the individual contribution h^n as a function (if possible) of N.

c) Now, suppose an all powerful and benevolent government took over society and decided to be the supplier of public health. This government can dictate the amount of contributions h_i individuals make. Suppose this benevolent government seeks to maximize the sum of individual utilities:

$$\sum_{i=1}^{N} u_i(x_i, H).$$

Solve the government's problem and find the optimal level of H^o as a function (if possible) of N. Compare your solution with the one from b. [Note that the technology for producing public health hasn't changed].

d) Suppose instead of dictating the amount of contributions each individuals make, the government sets up individual Lindahl prices. Find the level of price p that brings about the optimal solution from c.

3. Suppose three individuals must decide to produce or not one public good that cost \$150. Assume that the gross benefits from the public goods are respectively \$20, \$40, and \$100 for individuals 1, 2 and 3. If the good is produced, they each pay \$50. Each individual is asked to announce his own benefit for the public good, and the public good is produced only if the sum of reported benefits exceeds the total cost.

   a) Show that the Groves-Clarke tax induces truth-telling as a dominant strategy if each individual reports independently his own benefit.

   b) Show that the resulting provision of public good is optimal.

   c) What would be the decision if the provision of the public good was decided by majority vote, assuming that the cost per person is still the same? Compare your answer from part b.

4. There is a large number of commuters who decide to use their car or the subway. Commuting by subway takes 70 minutes whatever the number of commuters taking the subway. Commuting by car takes $C(x) = 20+60x$ minutes, where x is the proportion of commuters taking their cars, $0 \leq x \leq 1$.

   a) Plot the curves of the commuting time by car and the commuting time by subway as a function of the proportion of car users.

   b) What is the proportion of commuters who will take their car if everyone is taking her decision freely and independently so as ti minimize her own commuting time.

   c) What is the proportion of car users that minimizes the total commuting time. Compare this with your answer from part b. Interpret the difference. How large is the deadweight loss from the externality?

   d) Suppose that there are a 1000 commuters and that time is valued at 10 cents per minute, what would be the optimal toll imposed on commuters who use a car and explain why this would be beneficial for everyone.